RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY-JUNE 2013

FIRST YEAR

Statistics (General)

Paper : II

Date : 28/05/2013

Time : 11am – 1pm

Full Marks : 50

[Use separate Answer Books for each group]

Group-A

1. Answer **any three**:– 3×5
   - a) Prove that "correlation ratio" is less than or equal to 1. 5
   - b) Describe the method of fitting of an exponential curve by method of 'least-squares'. 5
   - c) Define Multiple & Partial correlation coefficient. Why multiple correlation coefficient is always greater than or equal to zero? 5
   - d) In case of linear regression, obtain the residual variance, var(e) and hence prove that $-1 \le r \le 1$. Also interpret the cases when $r = \pm 1$ from above. 4+1
   - e) Show that Spearman's Rank correlation coefficient is actually the simple product-moment correlation coefficient of the ranks. 5
   - f) What is scatter plot? Write down its uses. Prove that correlation-coefficient is independent of change of origin and scale. 1+1+3

2. Answer **any one**:– 1×10
   - a) i) Show that $0 \le r^2 \le e^2yx \le 1$, where symbols have their usual meanings. 4
     - ii) Determine the angle between two lines of regression. 3
     - iii) In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible:
       Variance of $X = 9$.
       Regression equations: $8X - 10Y + 66 = 0,\ 40X - 18Y = 214$.
       What are: A) the correlation coefficient between X & Y?
       B) the standard deviation of Y? 3
   - b) i) What is rank correlation coefficient? Deduce spearmen's rank correlation coefficient. 4+2
     - ii) 800 candidates of both sexes appeared at an examination. The boys outnumbered the girls by 15% of the total. The number of candidates who passed exceed the number of failed by 480. Equal number of boys and girls failed in the examination. Find the Yule's coefficient of association & comment. 4

Group-B

3. Answer **any three**:– 3×5
   - a) Define the distribution function (c.d.f) of a random variable. State the properties of it. 2+3
   - b) Let X and Y be iid random variables distributed uniformly on [0,1]. Find the pdf of $X + Y$. 5
   - c) Define the conditional distribution of a two-dimensional random variable. 5

     If $f(x,y) = \begin{cases} 2 - x - y; \text{ if } & 0 < x < 1 \\ & 0 < y < 1 \\ 0 \quad ; & \text{otherwise} \end{cases}$

     Find the conditional p.d.f. of X & Y. 2+3
   - d) State & prove Markov's inequality. 5
   - e) Define Pareto distribution.

     If 'm' be the median of a Pareto distribution, then prove that $m = 2^{\frac{1}{\alpha}} x_0$; where $x_0 > 0$ and $\alpha > 0$ (both are constants). 2+3

f) Let X and Y be independent random variables, each normally distributed with parameters (μ_1, λ_1) and (μ_2, λ_2) respectively. Use the Uniqueness Theorem of Moment-generating functions to find the distribution of $(2X-Y)$. 5

4. Answer **any <u>one</u>**:– 1×10

a) Let (X, Y) be jointly distributed with bivariate normal density with parameters $(\mu_x, \mu_y, \sigma_x, \sigma_y, \rho)$.

i) Write down the joint pdf of (X,Y). 2

ii) Find the marginal pdf of X. 2

iii) Calculate the median of Y. 2

iv) If X be a random variable having exponential distribution, then prove that

$P[X > a+b \mid x > a] = P[x > b]$, for any $a > 0$, $b > 0$.

What is the name of this special property? 3+1

b) i) Define "Convergence in Probability" of a sequence of random variables. 2

ii) Let, $\{X_n\}$ be a sequence of random variables with p.m.f.

$$P[X_n = 1] = \frac{1}{n},\ P[X_n = 0] = 1 - \frac{1}{n}$$

Show that, $X_n \xrightarrow{P} X$, where X is a random variable degenerate at '0'. i.e. $X_n \xrightarrow{P} 0$. 3

iii) What do you mean by "weak law of large numbers" of a sequence of random variables? 2

iv) Write a short note on Z-scaling. 3